A Robust Control with a Neural Network Structure for Uncertain Robot Manipulator

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A robust position control with the bound function of neural network structure is proposed for uncertain robot manipulators. The uncertain factors come from imperfect knowledge of system parameters, payload change, friction, external disturbance, and etc. Therefore, uncertainties are often nonlinear and time-varying. The neural network structure presents the bound function and does not need the concave property of the bound function. The robust approach is to solve this problem as uncertainties are included in a model and the controller can achieve the desired properties in spite of the imperfect modeling. Simulation is performed to validate this law for four-axis SCARA type robot manipulator.

Key Words : Robot Manipulator, Robust Control, NN(Neural Network), Lyapunov Stability, Bound Function

1. Introduction

Most industrial robots in the field adopt linear independent joint controllers for the position control although the dynamics of robots is highly nonlinear, e.g., due to the coupling among the link motions and the frictions in each joints.

To accomplish the task (the position control subject to the robot dynamic property) efficiently and accurately, a variety of manipulator controllers have been developed. Among them, simple PD or PID control schemes are the most popularly used. Surprisingly, these control schemes may asymptotically stabilize the robot in the sense of Lyapunov and show satisfactory performance for low-speed motion. However, since the coupling effects due to centrifugal, coriolis and gravity terms become active in high-speed motion, the PD or PID controllers are no longer acceptable for the high-speed and high-performance task. Furthermore, it's difficult to achieve the precise model for the robot manipulators and its environment. Namely, the dynamics of robot manipulator include uncertainties such as incorrect parameter, friction, varying payload and disturbances and etc. The robust approach is to solve this problem as uncertainties are introduced in a model and a controller is designed to achieve the desired properties in spite of the imperfect modeling. The deterministic robust control design of manipulators can be found in, e.g., Chen (1991), Chen and Pandey (1990), Reithmeier and Leitmann (1991), Shoureshi et al.(1987), Ha and Han (2000), Han et al.(1997) and their bibliographies (Ge et al., 1998, 2001, 2002). On the other hand, owing to the increase for interest of a model-free controller, many researches have been accomplished for Neural network. (Yesildirek et

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al., 1996, 1995, 1997)

In the previous work Ha and Han (2000), we proposed a class of adaptive robust control of robot manipulators and analyzed the stability in sense of Lyapunov. This controller does not need the information of bound for uncertain factors but must have a concave bound function. In this work, we propose a class of robust control with a neural network structure for robot manipulators and analyze the stability in sense of Lyapunov. This control law also does not need the bound information. Furthermore, this doesn't require a concave bound function. So, we expect that the result of this work is more broadly applicable. The simulation results of 4-axis SCARA type robot are provided to show the effectiveness of the proposed algorithm.

2. Robot Dynamic

The joint coordinate model of the N-link robot manipulator is derived from the Newton-Euler equations, which can be written as :

$$M(q,\beta)\ddot{q}+C(q,\dot{q},\beta)\dot{q}+g(q,\beta)+f(q,\dot{q},t,\beta)=\tau \quad (1)$$

$q: n\times 1$	joint position vector
$M(q,\beta): n\times n$	inertia matrix
$C(q,\dot{q},\beta)\dot{q}: n\times 1$	centrifugal and coriolis vector
$g(q,\beta): n\times 1$	gravity vector
$f(q,\dot{q},t,\beta): n\times 1$	friction force vector
$\tau: n\times 1$	torque vector
β	uncertain factors

Remark 1. The inertia matrix $M(q, \beta)$ is symmetric and positive definite. The nonlinear term $C(q, \dot{q}, \beta)$ in (1) can be suitably chosen such that $\dot{M}(q, \beta)-2C(q, \dot{q}, \beta)$ is skew symmetric. The $f(q, \dot{q}, t, \beta)$ include the torque that occur due to structure, non-structure uncertainty such as friction and disturbance.

Standard notations are adopted with vector norms being euclidean. Matrix norm is the corresponding induced norm. Thus, for a real matrix M, $\|M\|=\sqrt{\lambda_{\min}(M^TM)}$, $\lambda_{\min}(\cdot)$ $(\lambda_{\max}(\cdot))$ is the mini-mum(maximum) eigen value of the given matrix.

3. Neural Network

A neural network which has the three-layer structure is applied to a robot manipulator by sub-controller. The relationship of input-output is given as Fig. 1.

Figure 1 is described by

$$y(x)=\bar{W}^T\sigma(\bar{V}^T\bar{x}+\theta_v)+\theta_w \quad (2)$$

where,

$\bar{x}\in R^{n1}$	: Input vector
l	: The number of neuron
$p\in R^{n2}$	: Output vector
$\bar{W}\in R^{l\times n2}$	: A weight of two, three layer
$\bar{V}\in R^{n1\times l}$	: A weight of one, two layer
θ_w	: Ouput layer threshold
θ_v	: Hidden layer threshold
σ	: Activity function

The notation which has thresholds with weights is written as

$$\begin{aligned}&W[\bar{W}\theta_w]^T,\ V=[\bar{V}\theta_v]^T,\ x=[\bar{x}1]^T,\ \sigma=[\bar{\sigma}1]^T\\&y(x)=W^T\sigma(V^Tx)\end{aligned} \quad (3)$$

The tuning of weighting (W, V) includes the tuning of thresholds (θ_w, θ_v).

The estimation value of $y(x)$ is given as

$$\hat{y}(x)=\hat{W}^T\sigma(\hat{V}^Tx) \quad (4)$$

$\tilde{y}(x)$ is written by

$$\begin{aligned}\tilde{y}(x)&=y(x)-\hat{y}(x)\\&=W^T\sigma(V^Tx)-\hat{W}^T\sigma(\hat{V}^Tx)\end{aligned} \quad (5)$$

In the $\tilde{y}=y-\hat{y}$, $\sigma(V^Tx)$ is described by the taylor series expansion.

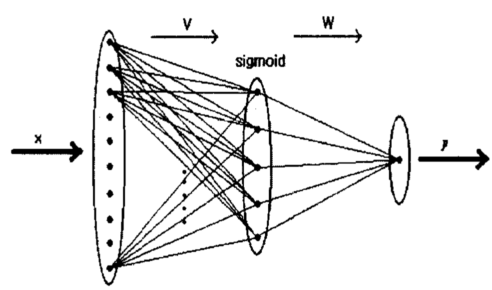

Fig. 1 Neural Network Structure

$$\tilde{y}(x)=\tilde{W}^T(\hat{\sigma}-\hat{\sigma}'\hat{V}^Tx)+\hat{W}^T\hat{\sigma}'\tilde{V}^Tx+w \quad (6)$$

where, $\hat{\sigma}=\sigma(\hat{V}^Tx)$

$$\hat{\sigma}'=\left.\frac{\partial\sigma(z)}{\partial z}\right|_{z=\hat{z}} \cong \mathrm{diag}\{\hat{\sigma}\}[I-\mathrm{diag}\{\hat{\sigma}\}]$$

Sub-error factor and bound can be written as

$$\begin{aligned} w(t)&=\tilde{W}^T\hat{\sigma}'\hat{V}^Tx+W^T(\ )(\hat{V}^Tx)^2+\epsilon_{th}(x) \\ \|w(t)\|&\leqq C_0+C_1\|\tilde{Z}\|_F+C_2\|x\|\|\tilde{Z}\|_F \end{aligned} \quad (7)$$

where, $Z\equiv\mathrm{diag}\{W,\ V\}$

C_0, C_1, C_2 : Computable constants

Assumption 1.

The weighting value$(W,\ V)$ has a bound value.

$$\|Z\|_F\leqq Z_m \quad (8)$$

$\|\cdot\|_F$: Fronius Norm

4. Control Law Design and Stability

The robust position control with the bound function of neural network structure is proposed for uncertain robot manipulators.

4.1 Control law design

The robust controller is given by

$$\begin{aligned} \tau=&\hat{M}(q)(\ddot{q}_d-S\dot{e})+\hat{C}(q,\ \dot{q})(\dot{q}_d-Se)+\hat{g}(q) \\ &+\hat{f}(q,\ \dot{q},\ t)+p(q,\ \dot{q},\ \hat{\rho},\ t)-K_ae-K_b\dot{e} \end{aligned} \quad (9)$$

The robust term $p(q,\ \dot{q},\ \hat{\rho},\ t)$ can be written as

$$p=\begin{cases} -\dfrac{\mu}{\|\mu\|}\hat{\rho}+v & \text{if}\|\mu\|\geqq\epsilon \\ -\dfrac{\mu}{\epsilon}\hat{\rho} & \text{if}\|\mu\|<\epsilon \end{cases} \quad (10)$$

$$\mu=(\dot{e}+Se)\hat{\rho}(q,\ \dot{q},\ t) \quad (11)$$

$$\rho(q,\ \dot{q},\ t)\geqq\|\Phi(q,\ \dot{q},\ \sigma,\ t)\| \quad (12)$$

$$\begin{aligned} \Phi(q,\ \dot{q},\ \beta,\ t)=&(\hat{M}-M)(\ddot{q}_d-S\dot{e})+(\hat{g}-g) \\ &+(\hat{C}-C)(\dot{q}_d-Se)+(\hat{f}-f) \end{aligned} \quad (13)$$

where, q_d : desired trajectory

$e=q-q_d$, $\dot{e}=\dot{q}-\dot{q}_d$

K_a, K_b : positive diagonal matrix.

The bound function(ρ) is following as

$$y=W^T\sigma(V^Tx)=\rho(q,\ \dot{q},\ t) \quad (14)$$

$$\hat{y}=\hat{W}^T\sigma(\hat{V}^Tx)=\hat{\rho}(q,\ \dot{q},\ t) \quad (15)$$

The neural network weighting value is given by

$$\begin{aligned} \dot{\hat{W}}&=2\|\dot{e}+Se\|\{F\sigma(\hat{V}^Tx)-F\hat{\sigma}'\hat{V}^Tx-\kappa F\hat{W}\} \\ &\qquad \text{if}\ \|\mu\|\geqq\epsilon \\ &=-2\|\dot{e}+Se\|\kappa F\hat{W} \\ &\qquad \text{if}\ \|\mu\|<\epsilon \end{aligned} \quad (16)$$

$$\begin{aligned} \dot{\hat{V}}&=2\|\dot{e}+Se\|\{Gx\hat{W}^T\hat{\sigma}'-\kappa G\hat{V}\} & \text{if}\|\mu\|\geqq\epsilon \\ &=-2\|\dot{e}+Se\|\kappa G\hat{V} & \text{if}\|\mu\|<\epsilon \end{aligned} \quad (17)$$

In the taylor series, the compensation for a high-order term is written by

$$v=-K_z(\|\hat{Z}\|+Z_m)(\dot{e}+Se) \quad (18)$$

where,

$\|Z\|_F\leqq Z_m$, $Z\equiv\mathrm{diag}\{W,\ V\}$, $K_z>C_2$

$F=F^T>0$, $G=G^T>0$, $\kappa>0$, $x=[q_d\dot{q}_d\ddot{q}_dSe\ \dot{e}]^T$

By using this robust control, we analyze the fact that the system (1) satisfies the stability; practical stability or global uniform attractivity. (Chen, 1991; Chen and Pandey, 1990; Corless and Leitmann, 1981)

4.2 Stability analysis

Definition 1. Practical stability for robot manipulator The closed-loop system is

$$\begin{aligned} M(q)\ddot{q}+C(q,\ \dot{q})\dot{q}+g(q)+f(q,\ \dot{q},\ t) \\ =\tau(q,\ \dot{q},\ \hat{\rho},\ t) \end{aligned} \quad (19)$$

$$q(t_0)=q_0,\ \dot{q}(t_0)=\dot{q}_0,\ \hat{\rho}(t_0)=\hat{\rho}_0$$

Consider a robust stabilizing control $\tau(q,\ \dot{q},\ \rho,\ t)$ (proved the pre-study) and $p(q,\ \dot{q},\ \rho,\ t)$ that satisfies assumption 1. If the system (1) is subject to the control $\tau(q,\ \dot{q},\ \hat{\rho},\ t)$ with $\hat{\rho}$ given in (15), then there exists a constant $\underline{d}>0$ such that the closed-loop system(19) has the following properties.

(a) Existence of solutions :

For each $(q_0,\ \dot{q}_0,\ \hat{\rho}_0,\ t_0)$, the system (1) possess a solution $(q(\cdot),\ \dot{q}(\cdot),\ \hat{\rho}(\cdot)): [t_0,\ t_1)\rightarrow R^n$.

(b) Uniform Boundedness :

Given r_1, $r_2\in[0,\ \infty)$, there exist $d_1(r_1\cdot r_2)$, $d_2(r_1,\ r_2)<\infty$ such that if $\|(q_0,\ \dot{q}_0)\|\leqq r_1$ and $\|\hat{\rho}_0-\rho\|\leqq r_2$, then $\|(q(t),\ \dot{q}(t))\|\leq d_1(r_1,\ r_2)$ and $\|\hat{\rho}(t)-\rho\|\leqq d_2(r_1,\ r_2)$, for all $t\in[t_0,\ \infty)$.

(c) Uniform Ultimate Boundedness :

Given any $\bar{d} > \underline{d}$ and any $r \in [0, \infty)$, there is a $T(\bar{d}, r) \in [0, \infty)$ such that if $\|(q_0, \dot{q}_0)\|$, $\|\hat{\rho}_0 - \rho\| \leq r$, then $\|(q(t), \dot{q}(t))\|$, $\|\hat{\rho}(t) - \rho\| \leq \bar{d}$, for all $t \geq t_0 + T(\bar{d}, r)$.

(d) Uniform Stability :

Given any $\bar{d} \geq \underline{d}$, there exists a $\delta(\bar{d}) > 0$ such that if $\|(q_0, \dot{q}_0)\|$, $\|\hat{\rho}_0 - \rho\| \leq \delta(\bar{d})$, then $\|(q(t), \dot{q}(t))\|$, $\|\hat{\rho}(t) - \rho\| \leq \bar{d}$, $t \geq [t_0, \infty)$.

This definition is the loose property of the asymptotic stability. Namely, this practical stability is what the trajectory of all states does not diverge but is maintained inside the ball round a circle center point in finite time.

Using the proof method of Corless and Leitmann (1981), we will show that the system (1) has a practical stability. Namely, the derivative of Lyapunov candidate function does always have a negative value beyond a ball which a circle center position is zero.

Lyapunov candidate function V is selected as

$$V = (\dot{e} + Se)^T M (\dot{e} + Se) + e^T (K_a + SK_b) e + \frac{1}{2} tr\{ \tilde{W}^T F^{-1} \tilde{W} \} + \frac{1}{2} tr\{ \tilde{V}^T G^{-1} \tilde{V} \} \tag{20}$$

The derivative of V is written by

$$\begin{aligned} \dot{V} = &2(\dot{e} + Se)^T M (\ddot{e} + S\dot{e}) \\ &+ (\dot{e} + Se)^T \dot{M} (\dot{e} + Se) + 2e^T (K_a + SK_b) \dot{e} \\ &+ tr\{ \tilde{W}^T F^{-1} \dot{\hat{W}} \} + tr\{ \tilde{V}^T G^{-1} \dot{\hat{V}} \} \end{aligned} \tag{21}$$

From (1) and (9), the following equation is derived.

$$\begin{aligned} M\ddot{e} &= M\ddot{q} - M\ddot{q}_d \\ &= \tau - C\dot{q} - g - f - M\ddot{q}_d \\ &= (\hat{M} - M)\ddot{q}_d - \hat{M}S\dot{e} + (\hat{C} - C)(\dot{q}_d - Se) \\ &\quad - C(\dot{e} + Se) + (\hat{g} - g) + (\hat{f} - f) \\ &\quad + p(q, \dot{q}, \hat{\rho}, t) - K_a e - K_b \dot{e} + v \end{aligned} \tag{22}$$

Substituting (21) into (22) and using the property of $\dot{M} - 2C$

$$\begin{aligned} \dot{V} = &2(\dot{e} + Se)^T \{ \Phi(q, \dot{q}, \sigma, t) + p(q, \dot{q}, \rho, t) \} \\ &- 2e^T SK_a e - 2\dot{e}^T K_b e \\ &+ 2(\dot{e} + Se)^T \{ p(q, \dot{q}, \hat{\rho}, t) - p(q, \dot{q}, \rho, t) \} \\ &+ tr\{ \tilde{W}^T F^{-1} \dot{\tilde{W}} \} + tr\{ \tilde{V}^T G^{-1} \dot{\tilde{V}} \} \end{aligned} \tag{23}$$

To avoid a complication of equation, (23) is divided by

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \tag{24}$$

$$\begin{aligned} \dot{V}_1 = &2(\dot{e} + Se)^T \{ \Phi(q, \dot{q}, \sigma, t) + p(q, \dot{q}, \rho, t) \} \\ &- 2e^T SK_a e - 2\dot{e}^T K_b e \end{aligned} \tag{25}$$

$$\begin{aligned} \dot{V}_2 = &2(\dot{e} + Se)^T \{ p(q, \dot{q}, \hat{\rho}, t) - p(q, \dot{q}, \rho, t) \} \\ &+ tr\{ \tilde{W}^T F^{-1} \dot{\tilde{W}} \} + tr\{ \tilde{V}^T G^{-1} \dot{\tilde{V}} \} \end{aligned} \tag{26}$$

From the previous work (2000), $\dot{V}_1$ is given as.

$$\begin{aligned} &\text{if } \|u\| \geq \epsilon \quad \dot{V}_1 \leq -R_1 \|\dot{e} + Se\|^2 \\ &\text{if } \|u\| < \epsilon \quad \dot{V}_1 \leq 2\epsilon - R_1 \|\dot{e} + Se\|^2 \\ &R_1 = \min\{ 2\kappa_{ai}/s_i,\ 2\kappa_{bi} \} \end{aligned} \tag{27}$$

Based on the value of $\|\mu\|$, the analysis of $\dot{V}_2$ is as follows.

Frist, if $\|\mu\| \geq \epsilon$, (10) subject to $\dot{V}_2$

$$p(q, \dot{q}, \hat{\rho}, t) = -\frac{\mu}{\|\mu\|} \hat{\rho}$$

and

$$\begin{aligned} \dot{V}_2 = &2\|\dot{e} + Se\| \{ \rho - \hat{\rho} \} + 2(\dot{e} + Se)^T v \\ &+ tr\{ \tilde{W}^T F^{-1} \dot{\tilde{W}} \} + tr\{ \tilde{V}^T G^{-1} \dot{\tilde{V}} \} \\ = &tr\{ \tilde{W}^T (F^{-1} \dot{\tilde{W}} + 2\|\dot{e} + Se\| (\hat{\sigma} - \hat{\sigma}' \hat{V}^T x)) \} \\ &+ tr\{ \tilde{V}^T (G^{-1} \dot{\tilde{V}} + 2\|\dot{e} + Se\| \hat{W}^T \hat{\sigma}' x) \} \\ &+ 2\|\dot{e} + Se\| w + 2(\dot{e} + Se)^T v \end{aligned} \tag{28}$$

Subject to (16), (17) and (18), $\dot{V}_2$ can be written as

$$\begin{aligned} \dot{V}_2 = &2\|\dot{e} + Se\| \kappa \{ tr\{ \tilde{W}^T (W - \tilde{W}) \} + \{ \tilde{V}^T (V - \tilde{V}) \} \} \\ &+ 2\|\dot{e} + Se\| w + 2\|\dot{e} + Se\|^2 (-K_z (\|\hat{Z}\|_F + Z_m)) \end{aligned} \tag{29}$$

where, $Z \equiv \text{diag}\{ W, V \}$ and $\|Z\|_F \leq Z_m$

The following equation can be in (29)

$$\begin{aligned} tr\{ \tilde{Z}^T (Z - \tilde{Z}) \} &= \langle \tilde{Z}, Z \rangle_F - \|\tilde{Z}\|_F^2 \\ &\leq \|\tilde{Z}\|_F \|Z\|_F - \|\tilde{Z}\|_F^2 \\ &\leq \|\tilde{Z}\|_F Z_m - \|\tilde{Z}\|_F^2 \end{aligned} \tag{30}$$

Assumption 2.

Q that satisfied $\|[q_d \dot{q}_d \ddot{q}_d]\| < Q$ exists.

From assumption 2., $\|x\| \leq \|\dot{e} + Se\| + Q$ and from (7),

$$\begin{aligned} \|w(t)\| &\leq C_0 + C_1 \|\tilde{Z}\|_F + C_2 \|\dot{e} + Se\| \|\tilde{Z}\|_F + C_2 Q \|\tilde{Z}\|_F \\ &\leq C_0 + C_1' \|\tilde{Z}\|_F + C_2 \|\dot{e} + Se\| \|\tilde{Z}\|_F \end{aligned} \tag{31}$$

Then, substituting (30) and (31) into (29),

$$\begin{aligned} \dot{V}_2 \leq &2\|\dot{e} + Se\| \cdot \kappa \cdot \|\tilde{Z}\|_F (Z_m - \|\tilde{Z}\|_F) \\ &- \|\dot{e} + Se\|^2 K_2 \|\hat{Z}\|_F + Z_m)) \\ &+ 2\|\dot{e} + Se\| (C_0 + C_1' \|\tilde{Z}\|_F + C_2 \|\dot{e} + Se\| \|\tilde{Z}\|_F) \end{aligned} \tag{32}$$

Now, using

$$\|\tilde{Z}\|_F \leqq \|Z\|_F + \|\hat{Z}\|_F \leqq Z_m + \|\hat{Z}\|_F,\ K_z > C_2$$

in (32),

$$\begin{aligned}&2\|\dot{e}+Se\|^2 C_2\|\tilde{Z}\|_F - 2K_z\|\dot{e}+Se\|^2(\|\hat{Z}\|_F + Z_m)\\ &\leqq 2\|\dot{e}+Se\|^2\{C_2(\|\hat{Z}\|_F+Z_m) - K_z(\|\hat{Z}\|_F+Z_m)\}\\ &\leqq 0\end{aligned} \tag{33}$$

Therefore, $\dot{V}_2$ is finally written as

$$\begin{aligned}\dot{V}_2 &\leqq -2\|\dot{e}+Se\|\{\kappa\|\tilde{Z}\|_F(-Z_m+\|\tilde{Z}\|_F)\\ &\quad - C_0 - C_1'\|\tilde{Z}\|_F\}\\ &\leqq -2\|\dot{e}+Se\|\{\kappa(\|\tilde{Z}\|_F - C_3)^2 - D\}\end{aligned} \tag{34}$$

Where, $C_3 = \frac{1}{2}Z_m + \frac{1}{2\kappa}C_1'$, $D = C_0 + \kappa C_3^2$

Second, if $\|\mu\| < \epsilon$, subject to (10)

$$p(q, \dot{q}, \hat{\rho}, t) = -\frac{\mu}{\epsilon}\hat{\rho}$$

and

$$\begin{aligned}\dot{V}_2 &= 2(\dot{e}+Se)^T\left\{\frac{\mu}{\epsilon}\rho - \frac{\mu}{\epsilon}\hat{\rho}\right\}\\ &\quad + tr\{\tilde{W}^T F^{-1}\dot{\tilde{W}}\} + tr\{\tilde{V}^T G^{-1}\dot{\tilde{V}}\}\\ &= 2\frac{\|\dot{e}+Se\|^2}{\epsilon}\{\rho^2 - \hat{\rho}^2\}\\ &\quad + tr\{\tilde{W}^T F^{-1}\dot{\tilde{W}}\} + tr\{\tilde{V}^T G^{-1}\dot{\tilde{V}}\}\end{aligned} \tag{35}$$

In (35),

$$\begin{aligned}&2\frac{\|\dot{e}+Se\|^2}{\epsilon}\{\rho^2 - \hat{\rho}^2\}\\ &= 2\epsilon\left(\left(\frac{\|\dot{e}+Se\|\rho}{\epsilon}\right)^2 - \left(\frac{\|\dot{e}+Se\|\hat{\rho}}{\epsilon}\right)^2\right)\\ &\leqq 2\epsilon\end{aligned} \tag{36}$$

From (16), (17), (18) and (30), $\dot{V}_2$ is written by

$$\begin{aligned}\dot{V}_2 &\leqq 2\epsilon + 2\kappa\|\dot{e}+Se\|(tr\{\tilde{W}(W-\tilde{W})\} + tr\{\tilde{V}(V-\tilde{V})\})\\ &\leqq 2\epsilon + 2\kappa\|\dot{e}+Se\|\|\tilde{Z}\|_F(Z_m - \|\tilde{Z}\|_F)\\ &\leqq 2\epsilon - 2\kappa\|\dot{e}+Se\|\left(\|\tilde{Z}\|_F - \frac{1}{2}Z_m\right)^2 + \frac{1}{4}\kappa\|\dot{e}+Se\|Z_m^2\end{aligned} \tag{37}$$

As we unite (27), (34) and (37), we have the following equation for $\dot{V}$

$$\text{if } \|\mu\| \geqq \epsilon \quad \begin{aligned}\dot{V} &\leqq -2\|\dot{e}+Se\|\{\kappa\|\tilde{Z}\|_F - C_3)^2 - D\}\\ &\quad - R_1\|\dot{e}+Se\|^2\end{aligned} \tag{38}$$

$$\text{if } \|\mu\| < \epsilon \quad \begin{aligned}\dot{V} &\leqq 4\epsilon - 2\kappa\|\dot{e}+Se\|\left(\|\tilde{Z}\|_F - \frac{1}{2}Z_m\right)^2\\ &\quad + \frac{1}{4}\kappa\|\dot{e}+Se\|Z_m^2 - R_1\|\dot{e}+Se\|^2\end{aligned} \tag{39}$$

Conclusively, in case $\|\mu\| \geqq \epsilon$ if $\|\tilde{Z}\|_F > C_3 + \sqrt{\frac{D}{\kappa}}$ or $\|\dot{e}+Se\| > \frac{D}{R_1}$ is satisfied then $\dot{V} < 0$. This fact shows that system (1) has a practical stability by proposed controller.

In case $\|\mu\| < \epsilon$ if $\|\dot{e}+Se\| > \frac{\kappa Z_m^2 + \sqrt{\kappa^2 Z_m^4 + 64\epsilon R_1}}{4R_1}$ is satisfied then $\dot{V} < 0$. Also, This fact shows that system (1) has a practical stability by proposed controller.

5. Simulation

We consider a 4-DOF SCARA type manipulator. In this simulation, we assume the lack of all masses knowledge and the lack of mass bound function. Hence, we shall treat these factor as the uncertainties.

Table 1 Robot specification and parameter

Mass (kg) \ Axis	1	2	3	4
Real	10.71	9.65	0.8	0.3
Normal	13	12	1.5	0.6

$\epsilon(=1.4)$, All elements of G, H are 1.
$Z_m = 10$, $\kappa = 1$, $K_z = [1\ 1\ 1\ 0.005]$

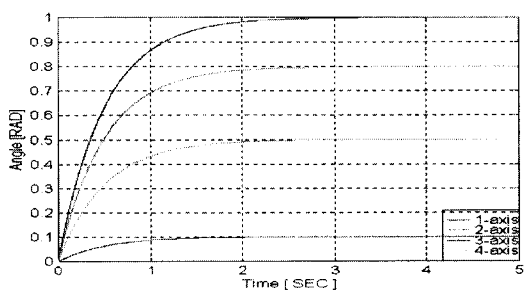

Fig. 2 Desired trajectory

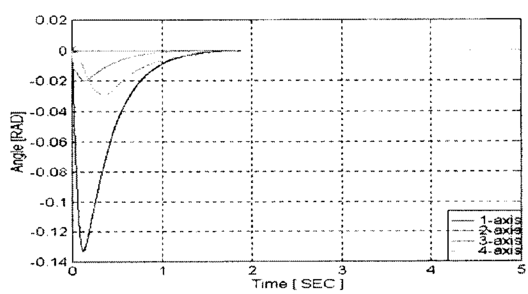

Fig. 3 Position error without uncertainties

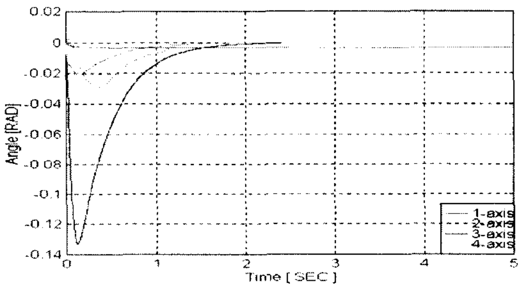

Fig. 4 Position error with uncertainties

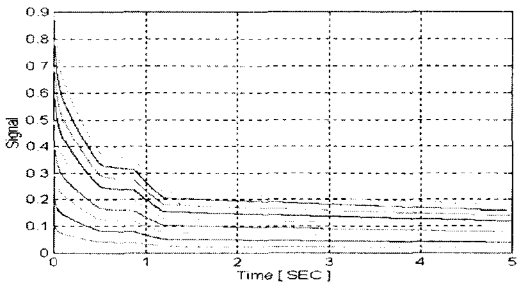

Fig. 5 Weighting value W (system with uncertainties)

The simulation results are shown graphically in Figs. 2-5.

In the Figs. 3-4, we know the effectiveness of the proposed algorithm. Figure 3 is a position error at a system without uncertainties and Fig. 4 is a position error at a system with uncertainties. In the Figs. 3 and 4, we know that the position error is nearly the same. Namely, the system with uncer tain factors has a practical stability by the proposed control law. Also, in the Fig. 5, the weighting value W is bounded and weighting value V is bounded.

6. Conclusion

For a robot manipulator, we proposed the robust controller with neural network structure and analyzed the stability in the sense of Lyapunov. Namely, the neural network estimate the bound function and the robust control use the estimated bound function. Unlike the previous work (2000), this control law don't need the concave property but the result of this control law is similar to the previous control law.

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